Appendix A
A dynamic macroeconomic model for fiscal policy analysis

Time is discrete and lasts forever, t = 0, 1, ..., ∞. The model with indivisible labor was pioneered by Hansen (1985)\(^1\) and Rogerson (1988).\(^2\) The economy is populated with households with identical preferences. Household utility is given by:

\[
U(C_t, G_t, N_t) = \phi_G \ln(C_t - bC_{t-1}) + (1 - \phi_G) \ln G_t + \xi_t \frac{(1 - N_t)^{1-\chi} - 1}{1 - \chi}
\]

The parameter \(\phi_G \in [0,1]\) measures the relative weights on private and government consumption. \(\xi_t\) is an exogenous stochastic variable governing the disutility from labor. The parameter \(\chi \neq 1\) governs the Frisch labor supply elasticity. Households either work or they do not. If they work, they work \(\bar{N}\) hours, with \(0 < \bar{N} < 1\). The amount of work, \(\bar{N}\), can be interpreted as a technological constraint and is exogenous to the model. Each period, there is a probability \(\pi_t\) of working \(0 < \pi_t < 1\). This probability is a choice variable – the household can choose whether to work but not how much it works. There is a lottery such that the household has a \(\pi_t\) likelihood of working. The rule of the game is that there is perfect insurance such that every household gets paid regardless of whether or not its members work. In expectations, households will work \(N_t = \pi_t \bar{N}\) and they will have identical consumption.

We can rewrite the utility function as:

\[
U(C_t, G_t, N_t) = \phi_G \ln(C_t - bC_{t-1}) + (1 - \phi_G) \ln G_t + \frac{N_t}{\bar{N}} \xi_t \frac{(1 - \bar{N})^{1-\chi} - 1}{1 - \chi} + \left(1 - \frac{N_t}{\bar{N}}\right) \xi_t \frac{(1)^{1-\chi} - 1}{1 - \chi}
\]

Which after some algebra becomes:

\[
U(C_t, G_t, N_t) = \phi_G \ln(C_t - bC_{t-1}) + (1 - \phi_G) \ln G_t + \frac{N_t}{\bar{N}} \xi_t \left(\frac{(1 - \bar{N})^{1-\chi} - 1}{1 - \chi} - \frac{(1 - \bar{N})^{1-\chi} - 1}{1 - \chi}\right) + \xi_t \frac{(1)^{1-\chi} - 1}{1 - \chi}
\]

Let \(\Gamma_t = \frac{\xi_t}{\bar{N}} \left(\frac{(1)^{1-\chi} - 1}{1 - \chi} - \frac{(1 - \bar{N})^{1-\chi} - 1}{1 - \chi}\right)\) and \(\Omega_t = \xi_t \frac{(1)^{1-\chi} - 1}{1 - \chi}\)

We can rewrite the household's utility function as:

\[
U(C_t, G_t, N_t) = \phi_G \ln(C_t - bC_{t-1}) + (1 - \phi_G) \ln G_t + \Gamma_t N_t + \Omega_t
\]

\(\Gamma_t\) is an exogenous shock to the preference for work that shifts employment decisions. The household discounts future utility flows by \(\beta \in (0,1)\). Households enter a period with a stock of government bonds \(B_t\) and a stock of physical capital \(K_{t-1}\). Households can save by accumulating more bonds or more capital. Capital \(K_t\) is leased to firms at rental rate \(R_t\).

Formally the household’s problem can be expressed:

\[
\max_{C_t, G_t, K_t, B_{t+1}, \beta, \omega, N_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, G_t, N_t)
\]

such that:
\[(1 + \tau_t^c)C_t + (1 + \tau_t^i)I_t + B_{t+1} - B_t \]
\[= \frac{1}{(1 - \tau_t^s n - \tau_t^f n)} w_t N_t + (1 - \tau_t^s k - \tau_t^f k) R_t K_{t-1} + \Pi_t + T_t + r_{t-1} B_t + M_t \]
\[K_{t+1} = I_t + (1 - \delta) K_t \]
\[\ln \Gamma_t = \rho_t \ln \Gamma_{t-1} + s_t \epsilon_{t, t} \]

$C_t, I_t, \Pi_t$ denote private consumption, investment in new physical capital and the lump sum profit resulting from the household ownership of firms. $T_t$ is a government transfer when positive and a lump sum tax when negative. Asset payments from abroad are denoted by $M_t$. The exogenous income or expenditure (when negative) captures a positive or negative trade balance, which introduces trade in a minimalist way. The trade balance influences the reaction of steady-state labor to tax changes. The household has to pay a tax on consumer goods $\tau_t^c$, a state tax on labor income $\tau_t^s n$, a federal tax on labor income $\tau_t^f n$, a state tax on investment income in comes $\tau_t^s k$, and a federal tax on investment income in comes $\tau_t^f k$. $\delta$ is the depreciation rate on physical capital.

The solution to the household's optimization problem satisfies:

\[\lambda_t = \frac{\phi_G}{(1 + \tau_t^c)(C_t - b C_{t-1})} - \beta b \mathbb{E}_t \phi_G (1 + \tau_{t+1}^c) (C_{t+1} - b C_{t})\]
\[\Gamma_t = \lambda_t (1 - \tau_t^s n - \tau_t^f n) w_t\]
\[\lambda_t (1 + \tau_t^i) = \beta \mathbb{E}_t [\lambda_{t+1} (1 - \tau_{t+1}^s k - \tau_{t+1}^f k) R_{t+1} + (1 + \tau_t^i) (1 - \delta)]\]
\[\lambda_t = \beta \mathbb{E}_t [\lambda_{t+1} (1 + r_t)]\]
\[K_{t+1} = I_t + (1 - \delta) K_t\]

with $\lambda_t$ as the Lagrange multiplier on the budget constraint.

Each firm uses capital services and labor to produce differentiated output $Y_t(j)$. Differentiated output is transformed into aggregate output $Y_t$ via the technology:

\[Y_t = \left[ \int_0^1 Y_t(j) \frac{\epsilon_p - 1}{\epsilon_p} dj \right]^{\epsilon_p / \epsilon_p - 1} \]

The parameter $\epsilon_p > 1$ is the elasticity of substitution among different differentiated goods. The retail sector solves the following profit maximization problem:

\[\max_{P_t(j)} \int_0^1 Y_t(j) \frac{\epsilon_p - 1}{\epsilon_p} dj - \int_0^1 P_t(j) Y_t(j) dj \]

$P_t(j)$ is the price charged for the output variety $j$, and $P_t$ is the aggregate price index. Profit maximization gives rise to the following downward demand curve for each variety of differentiated output:

\[Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon_p} Y_t \]

We can now solve for the price index. The nominal value of the final good is just the sum of the prices times quantities of intermediate goods. Using the above demand function:
\[ P_t Y_t = \int_0^1 P_t(j) Y_t(j) dj = \int_0^1 P_t(j) \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon_p} Y_t dj \]

\[ P_t Y_t = P_t^{\epsilon_p} Y_t \int_0^1 P_t(j)^{1-\epsilon_p} dj \]

\[ P_t^{1-\epsilon_p} = \int_0^1 P_t(j)^{1-\epsilon_p} dj \]

The intermediate goods firms (wholesalers) produce output using capital and labor, according to a standard production technology:

\[ Y_t(j) = \max\{ A_t K_{t-1}(j)^\alpha N_t(j)^{1-\alpha} - F, 0\} \]

where \( \ln A_t = (1 - \rho_A) \ln A + \rho_A \ln A_{t-1} + s_A \epsilon_{A,t} \)

It follows that aggregate capital and aggregate employment are just the sum of factor inputs across intermediate goods firms:

\[ K_{t-1} = \int_0^1 K_{t-1}(j) dj \]

\[ N_t = \int_0^1 N_t(j) dj \]

Capital is denoted by \( K_{t-1} \). \( A_t \) is an exogenous stochastic variable governing the level of aggregate productivity. It is common to all firms. Firms have market power. As such they are able to set their prices given that they face downward sloping demand curves. Hence they want to solve the following constrained problem:

\[ \max_{Y_t(j), P_t(j), K_{t-1}(j), N_t(j)} P_t(j) Y_t(j) - W_t N_t(j) - R_t K_{t-1}(j) \]

subject to:

\[ Y_t(j) = A_t K_{t-1}(j)^\alpha N_t(j)^{1-\alpha} \]

and

\[ Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon_p} Y_t \]

It is easy to show that we can think of there being an aggregate production function (for the retail good) that is identical to the production function of any intermediate goods firm:

\[ W_t = \frac{\epsilon_p - 1}{\epsilon_p} (1 - \alpha) A_t K_{t-1}^\alpha N_t^{-\alpha} \]

\[ R_t = \frac{\epsilon_p - 1}{\epsilon_p} (\alpha - 1) A_t K_{t-1}^\alpha N_t^{1-\alpha} \]

Since \( \frac{\epsilon_p^{-1}}{\epsilon_p} \leq 1 \), factors will be paid less than their marginal products giving rise to economic profits for the intermediate goods firms, \( \psi \equiv 1/\frac{\epsilon_p^{-1}}{\epsilon_p} \geq 1 \) is defined as the markup of price over the marginal cost. We assume that the log of the markup follows a stationary AR(1) process:
\[ \ln \psi_t = (1 - \rho) \ln \psi + \rho \ln \psi_{t-1} + s \psi \varepsilon_{\psi, t} \]

The state government sets state fiscal policy but takes federal tax rates as given. The flow budget constraint is given by:

\[ G_t + T_t + \tau_{t-1} B_t \leq B_{t+1} - B_t + \tau_{t} C_t + \tau^{s,k} R_t K_{t-1} + \tau^{s,n} w_t N_t \]

\( G_t \) denotes government consumption, and \( B_t \) denotes the stock of debt with which government enters a period. Expenditures are financed with tax collections or by issuing new debt.

The definition of equilibrium is standard. All budget constraints must hold with equality, which implies that debt, capital and labor markets must clear.

The aggregate resource constraint is:

\[ Y_t = C_t + I_t + G_t - M_t \]

**Model parameterization**

Using regional data on Illinois from the Bureau of Economic Analysis and data on state tax collections from the U.S. Census Bureau, the model is calibrated to replicate the average position of the Illinois economy between 2003 and 2015.

The characterization of the deterministic steady state of the model is of interest since the steady state facilitates the calibration of the model. This is because, to a first approximation, the deterministic steady state coincides with the average position of the model economy. In turn, matching average values of endogenous variables to their observed counterparts (e.g., matching observed average values of the labor share, the consumption shares, or the investment-to-output ratio) can reveal information about structural parameters that can be exploited in the calibration of the model.

Using the solution from the households’ and firms’ choice problems, the steady state equilibrium solves the following system of equations:

\[ W = \frac{1}{\psi} (1 - \alpha) AK^\alpha N^{-\alpha} \]

\[ R = \frac{1}{\psi} \alpha AK^{\alpha - 1} N^{1 - \alpha} \]

\[ \frac{1}{\beta} = \frac{[(1 - \tau^{s,k} - \tau^{f,k}) R + (1 + \tau^i)(1 - \delta)]}{(1 + \tau^i)} \]

\[ \frac{1}{\beta} = R^b \]

\[ Y = AK^\alpha N^{1 - \alpha} = C + \delta K + G - M \]

\[ \Gamma = \frac{\phi_G (1 - \tau^{s,n} - \tau^{f,n}) W}{(1 + \tau^c) C} \]
In Table 5, we hold all nonstate taxes constant and simulate only the change in the state tax burden.