Appendix A

A dynamic macroeconomic model for fiscal policy analysis

Time is discrete and lasts forever, $t = 0, 1, \dots, \infty$. The model with indivisible labor was pioneered by Hansen (1985)[1] and Rogerson (1988).[2] The economy is populated with households with identical preferences. Household utility is given by:

$$U(C_t, G_t, N_t) = \phi_G \ln(C_t - bC_{t-1}) + (1-\phi_G)\ln G_t + \xi_t \frac{(1-N_t)^{1-\chi} - 1}{1-\chi}$$

The parameter $\phi_G \in [0,1]$ measures the relative weights on private and government consumption. ξ_t is an exogenous stochastic variable governing the disutility from labor. The parameter $\chi \neq 1$ governs the Frisch labor supply elasticity. Households either work or they do not. If they work, they work $\bar{N}$ hours, with $0 < \bar{N} < 1$. The amount of work, $\bar{N}$, can be interpreted as a technological constraint and is exogenous to the model. Each period, there is a probability π_t of working $0 < \pi_t < 1$. This probability is a choice variable – the household can choose whether to work but not how much it works. There is a lottery such that the household has a π_t likelihood of working. The rule of the game is that there is perfect insurance such that every household gets paid regardless of whether or not its members work. In expectations, households will work $N_t = \pi_t \bar{N}$ and they will have identical consumption.

We can rewrite the utility function as:

$$U(C_t, G_t, N_t) = \phi_G \ln(C_t - bC_{t-1}) + (1-\phi_G)\ln G_t + \frac{N_t}{\bar{N}} \xi_t \frac{(1-\bar{N})^{1-\chi} - 1}{1-\chi} + \left(1 - \frac{N_t}{\bar{N}}\right) \xi_t \frac{(1)^{1-\chi} - 1}{1-\chi}$$

Which after some algebra becomes:

$$U(C_t, G_t, N_t) = \phi_G \ln(C_t - bC_{t-1}) + (1-\phi_G)\ln G_t + N_t \frac{\xi_t}{\bar{N}} \left(\frac{(1)^{1-\chi} - 1}{1-\chi} - \frac{(1-\bar{N})^{1-\chi} - 1}{1-\chi} \right) + \xi_t \frac{(1)^{1-\chi} - 1}{1-\chi}$$

Let $\Gamma_t = \frac{\xi_t}{\bar{N}} \left(\frac{(1)^{1-\chi} - 1}{1-\chi} - \frac{(1-\bar{N})^{1-\chi} - 1}{1-\chi} \right)$ and $\Omega_t = \xi_t \frac{(1)^{1-\chi} - 1}{1-\chi}$

We can rewrite the household's utility function as:

$$U(C_t, G_t, N_t) = \phi_G \ln(C_t - bC_{t-1}) + (1-\phi_G)\ln G_t + \Gamma_t N_t + \Omega_t$$

Γ_t is an exogenous shock to the preference for work that shifts employment decisions. The household discounts future utility flows by $\beta \in (0,1)$. Households enter a period with a stock of government bonds B_t and a stock of physical capital K_{t-1}. Households can save by accumulating more bonds or more capital. Capital K_t is leased to firms at rental rate R_t.

Formally the household's problem can be expressed:

$$\max_{C_t, I_t, K_{t+1}, B_{t+1}, w_t, N_t} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \{U(C_t, G_t, N_t)\}$$

such that:

$$
\begin{aligned}
(1+\tau_t^c)C_t + (1+\tau_t^i)I_t + B_{t+1} - B_t \\
= \left(1-\tau_t^{s,n}-\tau_t^{f,n}\right)w_t N_t + \left(1-\tau_t^{s,k}-\tau_t^{f,k}\right)R_t K_{t-1} + \Pi_t + \mathrm{T}_t + r_{t-1}B_t + M_t
\end{aligned}
$$

$$K_{t+1} = I_t + (1-\delta)K_t$$

$$ln\Gamma_t = \rho_\Gamma \ln \Gamma_{t-1} + s_\Gamma \varepsilon_{\Gamma,t}$$

C_t, I_t, Π_t denote private consumption, investment in new physical capital and the lump sum profit resulting from the household ownership of firms. T_t is a government transfer when positive and a lump sum tax when negative. Asset payments from abroad are denoted by M_t. The exogenous income or expenditure *(when negative)* captures a positive or negative trade balance, which introduces trade in a minimalist way. The trade balance influences the reaction of steady-state labor to tax changes. The household has to pay a tax on consumer goods τ_t^c, a state tax on labor income $\tau_t^{s,n}$, a federal tax on labor income $\tau_t^{f,n}$, a state tax on investment income $\tau_t^{s,k}$, and a federal tax on investment income $\tau_t^{f,k}$. δ is the depreciation rate on physical capital.

The solution to the household's optimization problem satisfies:

$$\lambda_t = \phi_G \frac{1}{(1+\tau_t^c)(C_t - bC_{t-1})} - \beta b \mathbb{E}_t\, \phi_G \frac{1}{(1+\tau_{t+1}^c)(C_{t+1} - bC_t)}$$

$$\Gamma_t = \lambda_t\left(1-\tau_t^{s,n}-\tau_t^{f,n}\right)w_t$$

$$\lambda_t(1+\tau_t^i) = \beta \mathbb{E}_t\left[\lambda_{t+1}\left(1-\tau_{t+1}^{s,k}-\tau_{t+1}^{f,k}\right)R_{t+1} + (1+\tau_{t+1}^i)(1-\delta)\right]$$

$$\lambda_t = \beta \mathbb{E}_t\left[\lambda_{t+1}(1+r_t)\right]$$

$$K_{t+1} = I_t + (1-\delta)K_t$$

with λ_t as the Lagrange multiplier on the budget constraint.

Each firm uses capital services and labor to produce differentiated output $Y_t(j)$. Differentiated output is transformed into aggregate output Y_t via the technology:

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} dj\right]^{\epsilon_p/\epsilon_p - 1}$$

The parameter $\epsilon_p > 1$ is the elasticity of substitution among different differentiated goods. The retail sector solves the following profit maximization problem:

$$\max_{Y_t(j)} P_t \left[\int_0^1 Y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} dj\right]^{\epsilon_p/\epsilon_p - 1} - \int_0^1 P_t(j) Y_t(j) dj$$

$P_t(j)$ is the price charged for the output variety j, and P_t is the aggregate price index. Profit maximization gives rise to the following downward demand curve for each variety of differentiated output:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} Y_t$$

We can now solve for the price index. The nominal value of the final good is just the sum of the prices times quantities of intermediate goods. Using the above demand function:

$$P_tY_t = \int_0^1 P_t(j)Y_t(j)dj = \int_0^1 P_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} Y_t dj$$

$$P_tY_t = P_t{}^{\epsilon_p} Y_t \int_0^1 P_t(j)^{1-\epsilon_p} dj$$

$$P_t{}^{1-\epsilon_p} = \int_0^1 P_t(j)^{1-\epsilon_p} dj$$

The intermediate goods firms (wholesalers) produce output using capital and labor, according to a standard production technology:

$$Y_t(j) = \max\{A_t K_{t-1}(j)^{\alpha} N_t(j)^{1-\alpha} - F, 0\}, 0 < \alpha < 1, F \geq 0$$

where $lnA_t = (1-\rho_A)lnA + \rho_A \ln A_{t-1} + s_A \varepsilon_{A,t}$

It follows that aggregate capital and aggregate employment are just the sum of factor inputs across intermediate goods firms:

$$K_{t-1} = \int_0^1 K_{t-1}(j)dj$$

$$N_t = \int_0^1 N_t(j)dj$$

Capital is denoted by K_{t-1}. A_t is an exogenous stochastic variable governing the level of aggregate productivity. It is common to all firms. Firms have market power. As such they are able to set their prices given that they face downward sloping demand curves. Hence they want to solve the following constrained problem:

$$\max_{Y_t(j), P_t(j), K_{t-1}(j), N_t(j)} P_t(j)Y_t(j) - W_t N_t(j) - R_t K_{t-1}(j)$$

subject to:

$$Y_t(j) = A_t K_{t-1}(j)^{\alpha} N_t(j)^{1-\alpha}$$

and

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} Y_t$$

It is easy to show that we can think of there being an aggregate production function (for the retail good) that is identical to the production function of any intermediate goods firm:

$$W_t = \frac{\epsilon_p - 1}{\epsilon_p}(1-\alpha)A_t K_{t-1}{}^{\alpha} N_t{}^{-\alpha}$$

$$R_t = \frac{\epsilon_p - 1}{\epsilon_p}\alpha A_t K_{t-1}{}^{\alpha-1} N_t{}^{1-\alpha}$$

Since $\frac{\epsilon_p-1}{\epsilon_p} \leq 1$, factors will be paid less than their marginal products giving rise to economic profits for the intermediate goods firms, $\psi \equiv 1/\frac{\epsilon_p-1}{\epsilon_p} \geq 1$ is defined as the markup of price over the marginal cost. We assume that the log of the markup follows a stationary AR(1) process:

$$ln\,\psi_t = (1-\rho_\psi) ln\,\psi + \rho_\psi \ln\ \psi_{t-1} + s_\psi \varepsilon_{\psi,t}$$

The state government sets state fiscal policy but takes federal tax rates as given. The flow budget constraint is given by:

$$G_t + T_t + r_{t-1}B_t \leq B_{t+1} - B_t + \tau_t^c C_t + \tau_t^{s,k} R_t K_{t-1} + \tau_t^{s,n} w_t N_t$$

G_t denotes government consumption, and B_t denotes the stock of debt with which government enters a period. Expenditures are financed with tax collections or by issuing new debt.

The definition of equilibrium is standard. All budget constraints must hold with equality, which implies that debt, capital and labor markets must clear.

The aggregate resource constraint is:

$$Y_t = C_t + I_t + G_t - M_t$$

Model parameterization

Using regional data on Illinois from the Bureau of Economic Analysis and data on state tax collections from the U.S. Census Bureau, the model is calibrated to replicate the average position of the Illinois economy between 2003 and 2015.

The characterization of the deterministic steady state of the model is of interest since the steady state facilitates the calibration of the model. This is because, to a first approximation, the deterministic steady state coincides with the average position of the model economy. In turn, matching average values of endogenous variables to their observed counterparts (e.g., matching observed average values of the labor share, the consumption shares, or the investment-to-output ratio) can reveal information about structural parameters that can be exploited in the calibration of the model.

Using the solution from the households' and firms' choice problems, the steady state equilibrium solves the following system of equations:

$$W = \frac{1}{\psi}(1-\alpha)AK^\alpha N^{-\alpha}$$

$$R = \frac{1}{\psi}\alpha AK^{\alpha-1}N^{1-\alpha}$$

$$\frac{1}{\beta} = \frac{\left[(1-\tau^{s,k}-\tau^{f,k})R + (1+\tau^i)(1-\delta)\right]}{(1+\tau^i)}$$

$$\frac{1}{\beta} = R^b$$

$$Y = AK^\alpha N^{1-\alpha} = C + \delta K + G - M$$

$$\Gamma = \frac{\phi_G(1-\tau^{s,n}-\tau^{f,n})W}{(1+\tau^c)C}$$

Table 3: Restrictions on model parameters

Fixed and estimated parameters

Parameters	Value	Restriction
Capital share of income	$\alpha = 0.33$	Bureau of Economic Analysis
Net flows from abroad	$\frac{m}{y} = -0.0675$	Implied from BEA data on consumption, investment and government expenditures
Preference weight on private consumption	$\phi_G = 0.8$	Bouakez and Rebei (2007)
Labor wedge		Investment share in GDP
(Inverse of the markup)	$\psi = 1.2912$	Chari, Kehoe and McGrattan (2008) estimate: 1.26 for U.S.

Note: Fixed parameters are parameters that are directly observed and taken from data observations.

Source: Bureau of Economic Analysis regional data and author's calculations **@illinoispolicy**

Table 4: Illinois tax rate data

Effective tax rate computations using the McDaniels (2007) methodology

	Average effective tax rates, baseline (2003-2015)	Simulated deviations (% change from base-line) consistent with average income tax revenues as a share of GDP from 2011-2015
Federal tax on labor income	14.10%	No change
Federal tax on investment/capital income	13.80%	No change
State tax on labor income	1.80%	23.30%
State tax on investment/capital income	3.40%	22.00%
State tax on consumer goods	3.10%	No change
State tax on investment goods	1.70%	No change
% of GDP paid in state income tax	**2.70%**	**19.30%**

Source: U.S. Census Bureau and author's calculations **@illinoispolicy**

In Table 5, we hold all nonstate taxes constant and simulate only the change in the state tax burden.

Table 5: Free parameters are parameters that are estimated – pinned down – by data observations

Depreciation rate of capital	$\delta=0.091$	Regional BEA data based on capital stock estimate
Labor supply parameter	$\Gamma=1.002$	Pinned down by the share of time spent at work $n=0.21$

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[1] Hansen, Gary D, "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics* 16, no. 3 (1985): 309-327.

[2] Rogerson, Richard, "Indivisible Labor, Lotteries and Equilibrium," *Journal of Monetary Economics* 21, no. 1 (1988): 3-16.