Appendix B: Fiscal policy in a neoclassical growth model

The basic model
Time is discrete and lasts forever. Consider a small open economy populated by a large number of identical infinitely lived households that aim to maximize lifetime utility by solving the following problem:

$$\max_{c_t,n_t,k_{t+1},b_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \ln(c_t) - \kappa n_t^{1+\varphi} + \omega(g_t)$$

with $\kappa > 0, k_0 > 0, \varphi > 0$

subject to:

$$(1 + \tau^0_t)c_t + x_t + b_{t+1} - b_t \leq (1 - \tau^{inc}_t)[w_t n_t + d_t k_{t-1}] + s_t + \pi_t + r_{t-1}b_t$$

$$k_t = x_t + (1 - \delta)k_{t-1}$$

where $c_t, n_t, x_t, b_t, k_t$ denote consumption of consumer goods, hours worked, investment, government bond, capital. Like Baxter and King (1993) or McGrattan (1994), it is assumed that government spending may be valuable only insofar as it provides utility separably from consumption and leisure. The households receive wages $w_t$, dividends $d_t$, profits $\pi_t$, from the firm if any. Transfers (tax or fee if negative) from government are denoted by $s_t$. The household has to pay a tax on consumer goods $\tau^0_t$ and an income tax $\tau^{inc}_t$.

The representative firm maximizes profits:

$$\max \zeta^t k^0_{t-1} n_t^{1-\theta} - d_t k_{t-1} - w_t n_t$$

where $\zeta^t$ denotes the trend of total factor productivity.

The state government faces the budget constraint:

$$g_t + r_{t-1}D_t = TR_t + D_{t+1} - D_t$$

This equation implies that government spending plus interest payments on existing debt $D_t$ cannot exceed tax revenue plus new debt issuance. We assume that the world interest rate is exogenous.

State government tax revenues are given by:

$$TR_t = \tau^s_t c_t + \tau^{inc}_t [w_t n_t + d_t k_{t-1}]$$
**The equilibrium**

A competitive equilibrium is a set of prices \( \{w_t, d_t, w_t\} \) and allocations \( \{c_t, k_{t+1}, n_t, b_{t+1}, D_{t+1}\} \) such that household and firm optimality conditions hold, the firms hire all the labor and capital supplied by the household, the household and firm budget constraints hold with equality and household bond-holdings equal government debt issuance in all periods.

**The deterministic steady state**

The characterization of the deterministic steady state is of interest for two reasons. First, the steady state facilitates the calibration of the model. This is because, to a first approximation, the deterministic steady state coincides with the average position of the model economy. In turn, matching average values of endogenous variables to their observed counterparts (e.g., matching predicted and observed average values of the labor share, the consumption shares) can reveal information about structural parameters that can be exploited in the calibration of the model. Second, the deterministic steady state is often used as a convenient point around which the equilibrium conditions of the stochastic economy are approximated (see Schmitt-Grohe and Uribe, 2003). For any variable, we denote its steady-state value by removing the time subscript.

The following system solves the household’s problem:

\[
\frac{1}{(1 + \tau_t^s)c_t} = \frac{\beta}{(1 + \tau_{t+1}^s)c_{t+1}}
\]

\[
\kappa (1 + \frac{1}{\phi}) n_t^\frac{1}{\phi} \frac{1}{(1 - \tau_t^{inc})w_t} = \frac{1}{(1 + \tau_t^s)c_t}
\]

\[
\frac{(1 + \tau_{t+1}^s)c_{t+1}}{\beta (1 + \tau_t^s)c_t} = [(1 - \tau_t^{inc})(d_{t+1}) + 1 - \delta]
\]

\[
\frac{(1 + \tau_{t+1}^s)c_{t+1}}{\beta (1 + \tau_t^s)c_t} = (1 + r_{t+1})
\]

The solution to the firm’s problem:

\[
d_t = \theta \xi_t^t k_{t-1}^{\theta - 1} n_t^{1-\theta}
\]

\[
w_t = (1 - \theta) \xi_t^t k_{t-1}^{\theta} n_t^{-\theta}
\]

Combining the household and firm’s problems implies:
\[
\frac{(1 + \tau_{t+1}^c)c_{t+1}}{\beta(1 + \tau_{t}^c)c_{t}} = (1 - \tau_{t+1}^{inc})(\theta \zeta^{t+1}k_t^{\theta-1}n_{t+1}^{1-\theta} - \delta) + \delta
\]

Since a stationary equilibrium implies \( k_t = k_{t+1} = k \), we write:

\[
\frac{1}{\beta} = (1 - \tau^{inc})(\theta \zeta \left(\frac{k}{n}\right)^{\theta-1}) + 1 - \delta
\]

We can then solve for the steady state capital to labor ratio:

\[
\frac{k}{n} = \left(\frac{(1 + r - 1 + \delta)}{(1 - \tau^{inc} \theta \zeta)}\right)^{1/(\theta - 1)}
\]

Given the share of time spent at work observed in the data, the above yields the steady state level of capital.

**Model baseline**
The depreciation rate of capital \( \delta \) and the world interest rate \( r \) are based on the average annual depreciation rate taken from the Bureau of Economic Analysis, \( \delta = 0.091 \) and \( r = 0.01 \).

The capital share \( \theta \) is set to match the observed average labor share. In the present model, the labor share is given by the ratio of labor income to output, which is \( 1 - \theta \) at all times.
## Model baseline

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Description</th>
<th>Restriction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^{inc}$</td>
<td>0.0240</td>
<td>Current effective income tax rate</td>
<td>COGFA</td>
</tr>
<tr>
<td>$\tau^{inc2}$</td>
<td>0.0246</td>
<td>Additional $635 million income tax rate</td>
<td>Estimated</td>
</tr>
<tr>
<td>$\tau^{inc2}$</td>
<td>0.0254</td>
<td>Additional $1.5 billion income tax rate</td>
<td>Estimated</td>
</tr>
<tr>
<td>$\tau^c$</td>
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<td>Sales tax rate</td>
<td>COGFA</td>
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<td>$C/Y$</td>
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<td>Consumption to GDP ratio</td>
<td>BEA</td>
</tr>
<tr>
<td>$I/Y$</td>
<td>0.212</td>
<td>Investment to GDP ratio</td>
<td>BEA</td>
</tr>
<tr>
<td>$G/Y$</td>
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<td>Government spending to GDP ratio</td>
<td>BEA</td>
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<tr>
<td>$N$</td>
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<td>Share of time spent in paid market work</td>
<td>BLS</td>
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<tr>
<td>$\chi$</td>
<td>15</td>
<td>Disutility of labor</td>
<td>Set to match hours worked</td>
</tr>
<tr>
<td>$r$</td>
<td>0.011</td>
<td>Avg. annual real interest rate (2003-2018)</td>
<td>FRED</td>
</tr>
<tr>
<td>$\delta$</td>
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<td>Annual depreciation rate of capital</td>
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</tr>
<tr>
<td>$\sigma$</td>
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<td>Elasticity of Labor Supply</td>
<td>Kimball and Shapiro (2003)</td>
</tr>
</tbody>
</table>

Note: BEA data represent long-run averages for 1963-2016.