Appendix B: Fiscal policy in a neoclassical growth model

The basic model

Time is discrete and lasts forever. Consider a small open economy populated by a large number of identical infinitely lived households that aim to maximize lifetime utility by solving the following problem:

$$\max_{c_t,n_t,k_{t+1},b_{t+1}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t ln(c_t) - \kappa n_t^{1+\frac{1}{\varphi}} + \omega(g_t)$$

with $\kappa > 0, k_o > 0, \varphi > 0$

subject to:

$$(1+\tau_t^s)c_t + x_t + b_{t+1} - b_t \leq \left(1-\tau_t^{inc}\right)[w_t n_t + d_t k_{t-1}] + s_t + \pi_t + r_{t-1}b_t$$

$$k_t = x_t + (1-\delta)k_{t-1}$$

where c_t, n_t, x_t, b_t, k_t, denote consumption of consumer goods, hours worked, investment, government bond, capital. Like Baxter and King (1993) or McGrattan (1994), it is assumed that government spending may be valuable only insofar as it provides utility separably from consumption and leisure. The households receive wages w_t, dividends d_t, profits π_t, from the firm if any. Transfers (tax or fee if negative) from government are denoted by s_t. The household has to pay a tax on consumer goods τ_t^s and an income tax τ_t^{inc}.

The representative firm maximizes profits:

$$\max_{k_{t-1},n_t} \zeta^t k_{t-1}^{\theta} n_t^{1-\theta} - d_t k_{t-1} - w_t n_t$$

where ζ^t denotes the trend of total factor productivity.

The state government faces the budget constraint:

$$g_t + r_{t-1}D_t = TR_t + D_{t+1} - D_t$$

This equation implies that government spending plus interest payments on existing debt D_t cannot exceed tax revenue plus new debt issuance. We assume that the world interest rate is exogenous.

State government tax revenues are given by:

$$TR_t = \tau_t^s c_t + \tau_t^{inc}[w_t n_t + d_t k_{t-1}]$$

The equilibrium

A competitive equilibrium is a set of prices $\{w_t, d_t, w_t\}$ and allocations $\{c_t, k_{t+1}, n_t, b_{t+1}, D_{t+1}\}$ such that household and firm optimality conditions hold, the firms hire all the labor and capital supplied by the household, the household and firm budget constraints hold with equality and household bond-holdings equal government debt issuance in all periods.

The deterministic steady state

The characterization of the deterministic steady state is of interest for two reasons. First, the steady state facilitates the calibration of the model. This is because, to a first approximation, the deterministic steady state coincides with the average position of the model economy. In turn, matching average values of endogenous variables to their observed counterparts (e.g., matching predicted and observed average values of the labor share, the consumption shares) can reveal information about structural parameters that can be exploited in the calibration of the model. Second, the deterministic steady state is often used as a convenient point around which the equilibrium conditions of the stochastic economy are approximated (see Schmitt-Grohe and Uribe, 2003). For any variable, we denote its steady-state value by removing the time subscript.

The following system solves the household's problem:

$$\frac{1}{(1+\tau_t^s)c_t} = \frac{\beta}{(1+\tau_{t+1}^s)c_{t+1}}$$

$$\frac{\kappa\left(1+\frac{1}{\varphi}\right)n_t^{\frac{1}{\varphi}}}{\left(1-\tau_t^{inc}\right)w_t} = \frac{1}{(1+\tau_t^s)c_t}$$

$$\frac{(1+\tau_{t+1}^s)c_{t+1}}{\beta(1+\tau_t^s)c_t} = \left[\left(1-\tau_{t+1}^{inc}\right)(d_{t+1}) + 1 - \delta\right]$$

$$\frac{(1+\tau_{t+1}^s)c_{t+1}}{\beta(1+\tau_t^s)c_t} = (1+r_{t+1})$$

The solution to the firm's problem:

$$d_t = \theta\zeta^t k_{t-1}^{\theta-1} n_t^{1-\theta}$$

$$w_t = (1-\theta)\zeta^t k_{t-1}^{\theta} n_t^{-\theta}$$

Combining the household and firm's problems implies:

$$\frac{(1+\tau_{t+1}^{s})c_{t+1}}{\beta(1+\tau_{t}^{s})c_{t}} = \left(1-\tau_{t+1}^{inc}\right)\left(\theta\zeta^{t+1}k_{t}^{\theta-1}n_{t+1}^{1-\theta}-\delta\right)+\delta$$

Since a stationary equilibrium implies $k_t = k_{t+1} = k$, we write:

$$\frac{1}{\beta} = \left(1-\tau^{inc}\right)\left(\theta\zeta\left(\frac{k}{n}\right)^{\theta-1}\right)+1-\delta$$

We can then solve for the steady state capital to labor ratio:

$$\frac{k}{n} = \left(\frac{(1+r-1+\delta)}{(1-\tau^{inc})\theta\zeta}\right)^{1/(\theta-1)}$$

Given the share of time spent at work observed in the data, the above yields the steady state level of capital.

Model baseline

The depreciation rate of capital δ and the world interest rate r are based on the average annual depreciation rate taken from the Bureau of Economic Analysis, $\delta = 0.091$ and $r = 0.01$.

The capital share θ is set to match the observed average labor share. In the present model, the labor share is given by the ratio of labor income to output, which is $1-\theta$ at all times.

Model baseline

Variable	Value	Description	Restriction
τ^{inc}	0.0240	Current effective income tax rate	COGFA
τ^{inc1}	0.0246	Additional $635 million income tax rate	Estimated
τ^{inc2}	0.0254	Additional $1.5 billion income tax rate	Estimated
τ^{c}	0.01	Sales tax rate	COGFA
C/Y	0.607	Consumption to GDP ratio	BEA
I/Y	0.212	Investment to GDP ratio	BEA
G/Y	0.107	Government spending to GDP ratio	BEA
N	0.20	Share of time spent in paid market work	BLS
χ	15	Disutility of labor	Set to match hours worked
r	0.011	Avg. annual real interest rate (2003-2018)	FRED
δ	0.091	Annual depreciation rate of capital	BEA
σ	1	Elasticity of Labor Supply	Kimball and Shapiro (2003)

Note: BEA data represent long-run averages for 1963-2016.

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