Appendix

Tax revenues in time $t$ are:

$$TR_{i,t} = I_{i,t} \tau_{i,t}$$

Where $I_{i,t}$ is income of individual $i$ in time $t$ and $\tau_{i,t}$ is the tax rate of that individual in time $t$.

Tax revenues in time $t + 1$ are:

$$TR_{i,t+1} = I_{i,t+1} \tau_{i,t+1}$$

Where $I_{i,t+1} = (1 + \alpha)I_{i,t}$ is income of individual $i$ in time $t + 1$ if income grows at rate $\alpha$.

This implies that a change in tax revenues under a flat tax with no changes to the tax code ($\tau_{t+1} = \tau_t$) is simply:

$$\Delta Flat \equiv TR_{i,t+1} - TR_{i,t} = I_{i,t+1} \tau_{i,t+1} - I_{i,t} \tau_{i,t} = (I_{i,t+1} - I_{i,t}) \tau_{i,t} = \alpha I_{i,t} \tau_{i,t}$$

Under a graduated income tax, a small increase in income induces an increase in the tax rate faced by individual $i$ such that ($\tau_{t+1} > \tau_t$):

$$\Delta Prog \equiv TR_{i,t+1} - TR_{i,t} = I_{i,t+1} \tau_{i,t+1} - I_{i,t} \tau_{i,t} = I_{i,t} \left[ \tau_{i,t+1}(1 + \alpha) - \tau_{i,t} \right]$$

Therefore, relative to a flat tax, the change in tax revenue under a graduated income tax scheme is:

$$\frac{\Delta Prog}{\Delta Flat} = \frac{\left[ \tau_{i,t+1}(1 + \alpha) - \tau_{i,t} \right]}{\alpha \tau_{i,t}} > 1$$