Appendix

Tax revenues in time t are:

$$TR_{i,t} = I_{i,t}\tau_{i,t}$$

Where $I_{i,t}$ is income of individual i in time t and $\tau_{i,t}$ is the tax rate of that individual in time t.

Tax revenues in time $t+1$ are:

$$TR_{i,t+1} = I_{i,t+1}\tau_{i,t+1}$$

Where $I_{i,t+1} = (1+\alpha)I_{i,t}$ is income of individual i in time $t+1$ if income grows at rate α .

This implies that a change in tax revenues under a flat tax with no changes to the tax code ($\tau_{t+1} = \tau_t$) is simply:

$$\Delta Flat \equiv TR_{i,t+1} - TR_{i,t} = I_{i,t+1}\tau_{i,t+1} - I_{i,t}\tau_{i,t} = \big(I_{i,t+1} - I_{i,t}\big)\tau_{i,t} = \alpha I_{i,t}\tau_{i,t}$$

Under a graduated income tax, a small increase in income induces an increase in the tax rate faced by individual i such that ($\tau_{t+1} > \tau_t$):

$$\Delta Prog \equiv TR_{i,t+1} - TR_{i,t} = I_{i,t+1}\tau_{i,t+1} - I_{i,t}\tau_{i,t} = I_{i,t}\left[\,\tau_{i,t+1}(1+\alpha) - \tau_{i,t}\right]$$

Therefore, relative to a flat tax, the change in tax revenue under a graduated income tax scheme is:

$$\frac{\Delta Prog}{\Delta Flat} \equiv \frac{\left[\,\tau_{i,t+1}(1+\alpha) - \tau_{i,t}\right]}{\alpha\tau_{i,t}} > 1$$