Appendix B

Flat to progressive income taxes: predictions from standard neoclassical growth theory

The Model

We investigate the potential impact of the tax increases on Illinois' economy through the lens of neoclassical growth theory. The neoclassical growth model, first introduced by Ramsey (1928), is the most widely taught model of capital accumulation and long-run growth and is the workhorse of modern growth theory.

For example, see the popular graduate-level textbooks by Romer (2001) and Barro and Sala-i-Martin (1999). This model is also widely used for thinking about issues in public finance (Chamley 1986; Judd 1985).

Here we use the neoclassical growth model to estimate the potential impact of a tax increase dedicated for paying down the unfunded pension liability.

Economic environment

Households

Consider a closed economy populated by a large number of infinitely-lived heterogenous N workers. Each worker is indexed by her earning potential j. The share of j workers is denoted $\mu_j \in (0,1)$. There's a common discount factor β. Each worker receives income from previous savings and from labor.

Each worker has preferences for consumption and leisure, which are represented by the utility function $u_{j,t} = u(c_{j,t}, l_{j,t})$ where c_j is consumption, $l_j = \bar{T} - h_j$ leisure and h_j is the time spent in market work. The usual assumptions apply to the utility function. It is increasing in each argument, twice differentiable and strictly concave. It is additively separable and fulfills the Inada conditions. Given a fiscal policy π, the problem of a worker will be to choose c and l such as to maximize lifetime utility subject to a budget constraint.

$$U_{j,t}(\pi) \equiv \max_{\{c_{j,t}, l_{j,t}\}} \sum_{t=0}^{\infty} \beta^t \, u\big(c_{j,t}, l_{j,t}\ \big) \quad (1)$$

Such that

$$c_{j,t} + a_{j,t+1} = w_{j,t} h_{j,t} + (1 + r_t) a_{j,t} \quad (2)$$

$a_{j,t}$ denotes the total asset holdings by a type j worker. Initial asset holdings $a_{j,0}$ are taken as given. The budget constraint (2) expresses that individuals allocate their income, composed of labor and (gross) interest income net of taxes, to consumption and saving.

Technology and feasibility

There is a large number of competitive firms that have access to a production technology given by:

$$q_t = z{h_t}^{\alpha}{k_t}^{\gamma}$$

Where K represents the total amounts of capital supplied to the production of output, $0 < \gamma < \alpha, \alpha + \gamma \leq 1$. Since spot factor markets are assumed to be competitive, wages are given by the marginal productivities of each type of labor and return on capital is given by its marginal product implying:

$$z\gamma {h_t}^{\alpha}{k_t}^{\gamma-1} = \bar{r}_t - \delta$$

$$z\alpha {h_t}^{\alpha-1}{k_t}^{\gamma}\mu_j\theta_j = \bar{w}_j$$

Let $h_t = \sum_{j=1}^{J} \mu_j \theta_j h_{j,t}$ with $\sum_{j=1}^{J} \mu_j = 1$ and $0 < \theta_1 < \theta_2 \ldots < \theta_J < 1$ represent the total supply of effective labor used in the production of output. $\frac{\theta_2}{\theta_1}$ denotes the relative productivity of workers of ability type 2. The depreciation rate of capital is denoted δ with $0 < \delta < 1$.

Feasibility requires the total (private and public) consumption plus investment to be less than or equal to aggregate output

$$c_t + k_{t+1} - (1-\delta)k_t + g_t \leq q_t \ (3)$$

Where c_t and g_t denote private and government consumption at date t. Note that aggregate consumption is obtained by adding up the weighted consumption of all individuals at date t:

$$c_t = \sum_{j=1}^{J} \mu_j c_{j,t} \ \ (4)$$

And aggregate investment is: $k_{t+1} - (1-\delta)k_t = \sum_{j=1}^{J} \mu_j a_{j,t+1}$ (5)

The government

Following Lansing and Guo (1998), the government balances its budget at each point in time and chooses a tax code summarized by the tax rate, $\pi = \tau(y, q)$, where y denotes household income and q is aggregate income. Thus, the tax rate which applies to a given household depends on its standing in the economy. This modeling assumption ensures that not all households eventually face the highest marginal tax rate simply as a result of economic growth. In the analysis, we further assume that government sets $\tau(y, q)$ according to the following tax schedule,

$$\tau_j\left(\frac{y}{q}\right) = \varsigma_j\left(\frac{y_j}{q}\right)^{\phi_j}$$

with $0 \leq \varsigma_j < 1, \phi_j > 0$.

The parameters ς, ϕ determine the level and the slope of the tax schedule respectively. When $\phi > 0$ households with higher taxable income are subject to higher tax rates, and the common case of proportional taxes corresponds to $\phi_j = 0, \tau\left(\frac{y}{q}\right) = \varsigma_j$. In making decisions about how much to consume and invest, households will take into account the particular way in which the tax schedule affects their earnings. Given the tax rate faces by a household, $\tau\left(\frac{y}{q}\right)y$ represents the total amount of taxes paid by a household with income y. Because we wish to draw the implications of progressivity on economic outcomes, it is helpful to distinguish between average and marginal tax rates. In this case, as taxable income changes, total taxes paid evolve according to

$$\frac{\partial\left[\tau\left(\frac{y}{q}\right)y\right]}{\partial y}=\tau_{m,j}\frac{y_j}{q}=(1+\phi_j)\varsigma_j\left(\frac{y_j}{q}\right)^{\phi_j}$$

where $\tau_m\frac{y}{q}$ is the tax rate applied to the last dollar earned. The average tax rate is simply $\tau\left(\frac{y}{q}\right)$. While there exists no single "appropriate" way to define the degree of progressivity of a tax schedule, one of the more widely used definitions is expressed in terms of the ratio of the marginal to the average tax rate. Specifically, a tax schedule is said to be progressive whenever the marginal rate exceeds the average rate at all levels of income. In our setup the ratio of the marginal rate to the average rate is simply $1+\phi_j$, so that the parameter ϕ_j captures the degree of progressivity in the tax code. In the limit, where $\phi_j=0$, the tax schedule is flat and $\tau_m\frac{y}{q}=\tau\left(\frac{y}{q}\right)$.

Total tax revenues are simply used to finance government expenditures on goods and services. The government budget constraint is then given by:

$$g_t=\sum_{j=1}^{J}(\bar{r}_t-r_{j,t})\,\mu_j a_{j,t}+\sum_{j=1}^{J}(\bar{w}_{j,t}-w_{j,t})\,\mu_j h_{j,t}\ (6)$$

Where $w_{j,t}\equiv\left(1-\varsigma_j\left(\frac{y_{j,t}}{q_t}\right)^{\phi_j}\right)\bar{w}_{j,t}$ and $r_t\equiv\left(1-\varsigma_j\left(\frac{y_{j,t}}{q_t}\right)^{\phi_j}\right)\bar{r}_t$ and $y_{j,t}=w_{j,t}h_{j,t}+(1+r_t)a_{j,t}$.

Equation (6) expresses that the government pays its expenditures by taxing wage income and interest income.

Steady state equilibrium

Given the tax environment $\{\varsigma_j,\phi_j\}$, the equilibrium allocations $\{\widehat{c_j},\widehat{a_j},\widehat{h_j},\widehat{y_j},\widehat{q},\widehat{k},\widehat{h},\ \widehat{c},\}$ and prices $\{\widehat{w_j},\hat{r}\}$ are such that workers maximize utility, the firm solves her maximization problem and the resource constraint holds with equality and that these allocations are constant for every period, i.e. $k_{t+1}=k_t=\hat{k}$.

Model parameterization and numerical examples

We choose parameter values from observed Illinois data to estimate the baseline economy. The parameters are presented in the table below.

Baseline parameters	Description	Data source
$\varsigma = 0.0495$	Level of the tax schedule	Illinois flat tax
ϕ=0	Slope of the tax schedule	Illinois flat tax
$\delta = 0.10$	Depreciation rate	Bureau of Economic Analysis
$\beta = 0.96$	Discount factor	Long-run average real interest rate (at annual frequency)
γ=0.3, $\alpha = (1 - \gamma)$	Capital share of income	Bureau of Economic Analysis
$\rho_1 = 0.30, \rho_2 = 0.57$	Labor supply elasticity parameter	Gruber and Saez (2002)
$\kappa = 0.18$	Disutility of labor parameter	Chosen to match average hours worked per worker
$g = 0.097$	Government share of GDP	Bureau of Economic Analysis
$\mu_1 = 0.129, \mu_2 = 0.139, \mu_3 = 0.151, \mu_4 = 083, \mu_5 = 0.040, \mu_6 = 0.033\ , \mu_7 = 0.006, \mu_8 = 0.001, \mu_9 = 0.0004$	Distribution of joint tax returns	IRS data for the distribution of tax returns
$\mu_{10} = 0.126, \mu_{11} = 0.138, \mu_{12} = 0.154, \mu_{13} = 086, \mu_{14} = 0.042, \mu_{15} = 0.036\ , \mu_{16} = 0.007, \mu_{17} = 0.001, \mu_{18} = 0.0004$	Distribution of single tax returns	IRS data for the distribution of tax returns
$\theta_1 = 1, \frac{\theta_2}{\theta_1} = 4.16, \frac{\theta_3}{\theta_1} = 8.4, \frac{\theta_4}{\theta_1} = 12.74, \frac{\theta_5}{\theta_1} =$	Average earnings per return in group j relative to group $j = 1$ earnings	IRS data for the distribution of income

$16.85, \frac{\theta_6}{\theta_1} = 24.92, \frac{\theta_7}{\theta_1} = 51.29, \frac{\theta_8}{\theta_1} = 121.29, \frac{\theta_9}{\theta_1} = 552.31$		