Equilibrium search model of the housing market

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In what follows, we describe an equilibrium search model of the housing market as in Wheaton (1990). The model is extended to include state and local taxes and federal income tax deductibility of state and local taxes.

We consider an economy endowed with a continuum of individuals. Time is discrete. Agents are infinitely lived and discount future income with a discount factor β. The economy consists of households that rent, households that are in the market to purchase a home, homeowners and "mismatched" households – sellers. All agents are risk neutral. Households are either renters, homeowners, sellers or buyers searching for a home. All housing units are assumed to be identical and have the same value.

At the beginning of the period a fraction δ of all homeowners are mismatched and become sellers. Further, there are search frictions in the housing market, meaning that it takes time for buyers to find a suitable house and for the seller to find a buyer for a house.

The number of matches in a given period is given by a matching function $M(b,v)$, where b and v are the measure of buyers and vacancies. Assume that the matching function is concave, increasing in both of its arguments and displays constant returns to scale. Given this matching function, buyers find a suitable home with probability $p^d = m(\theta) \equiv \frac{M(b,v)}{b} = M(1,1/\theta)$ and the realtor finds a suitable home with probability p^s=$\theta m(\theta) = \frac{M(b,v)}{(1-\delta)v} = M(\theta,1)$ where θ denotes the housing market tightness and is given by the ratio of buyers to vacancies $\theta \equiv b/v$.

Households derive utility ε when they own a house, whereas buyers searching for a house derive utility u^u, with $u^u < u^m$. In addition, households are endowed with y=$w(1-\tau^s-\tau^y+\tau^y\gamma\tau^s)$ where w is a wage set exogenously and households must pay state and federal taxes τ^s and τ^y respectively. They get some of those taxes back if $\gamma > 0$ (deduction of state taxes in the federal income tax liability).

The Bellman equations for homeowners and buyers searching for a house are as follows.

The value to a buyer of actively searching for a new home is:

$$V^b[1-\beta(1-p^d)] = u^u + y - R - \kappa + \beta p^d V^o - \beta p^d P \qquad Eq.1$$

Where:

$$y = w(1-\tau^s-\tau^y+\tau^y\gamma\tau^s)$$

A buyer receives utility flow $u^u + y$ pays the rent R and pays a search cost κ each period she fails to find a suitable house. A buyer finds a suitable house with probability p^d and once the match occurs, the buyer must pay the sales price P.

The value to a homeowner is:

$$V^o[1-\beta(1-\delta)] = u^u + y - m - P\left(\tau_p - w\tau^y\gamma\tau_p\right) + \beta\delta V^s \qquad Eq. 2$$

(Eq. 2) captures the dividends from owning a home. With probability δ the homeowner becomes "mismatched." If a homeowner enjoys his house, the homeowner receives utility flow $u^u + y$ and pays a maintenance cost m as well as the property tax bill net of what his deducted from his federal income tax liability. If there's a separation shock, the homeowner becomes a seller (Eq. 3).

The value to a seller is:

$$V^s[1-\beta(1-p^s)] = u^u + y - m - P\left(\tau_p - w\tau^y\gamma\tau_p\right) + \beta p^s V^{rs} + \beta p^s P \quad Eq. 3$$

A seller is on the market until she finds a buyer which occurs with probability p^s. If successful, the seller collects the sales price of the house P and becomes a renter.

The value of renter, who has owned in the past:

$$V^{rs}[1-\beta] = u^m + y - R + P \qquad Eq. 4$$

We assume that owning a house and then selling it is an absorbing state.

Rental contracts are signed in a Walrasian market (i.e renters and landlords take prices as given and the market clearing price is the one that equates supply and demand for rental units). The rent R covers homeowners' costs – the maintenance cost and property taxes so a homeowner would get the same utility flow from renting his own home as from occupying the same unit.

The population evolves according to:

$$N_{t+1}^b - N_t^b = \alpha N_t^R - p^s N_t^s$$

$$N_{t+1}^o - N_t^o = p^d N_t^b - \delta N_t^0$$

$$N_{t+1}^s - N_t^s = \delta N_t^0 - p^s N_t^s$$

In steady state the flow into each state must be equal to the flow out of that state. We remove the time subscript to denote steady state values:

$$\alpha N^r = p^s N^s \qquad Eq. 5$$

$$p^d N^b = \delta N^0 \qquad Eq. 6$$

$$\delta N^0 = p^s N^s \qquad Eq. 7$$

$$1 = N^r + N^b + N^o + N^s \quad Eq. 8$$

As is common in markets with search frictions, matching in the housing market leads to a surplus that must be shared among the buyer and the seller. As in Pissarides (2000), we assume that the house price P is determined by Nash Bargaining as in Nash (1950) and Rubinstein (1982). The house price P is the solution to a Nash Bargaining problem which delivers the following sharing rule, where χ is the share of the surplus going to buyers:

$$(1-\chi)(V^o - V^b) = \chi(V^{rs} - V^s) \qquad Eq.9$$

The equilibrium price is a weighted average between the present discounted value of net flows from owning a home and the benefit of selling that home.

Equilibrium

A search equilibrium is a list of value functions, measures of households, price (and rent) and probabilities such that $V^b = 0$[1] and equations (1-9) are satisfied.

Baseline calibration and application of the theory

The model is calibrated at a quarterly frequency. Following Ngai and Tenreyro (2014), the discount factor matches an annual interest rate of 6% and the bargaining power is set to χ =0.5. We set $\delta = 0.024$ so the average tenure is 9 years, as in Diaz and Jerez (2013). We impose a standard Cobb-Douglas matching function $m(\theta) = \min\{\mu\theta^{-\eta}, 1\}$. We follow Genesove and Han (2012) and set $\eta = 0.16$. The tax rate are $\tau_p = 0.021$, set to match the average effective property tax rate, $\tau_y = 0.15$ is set to match the average effective federal income tax rate (the use of the effective rate implies $\gamma = 0$), and $\tau_s = 0.0495$ is set to match the average effective state income tax rate.

We set $\kappa = 12.5923$ and $\mu = 0.5$ are jointly calibrated to match the average market tightness $\theta = 1.006$ and the share of homeowners in the population ($N^0 = 0.59$) The number of rental units $U = 0.4496$ is set to match the rent to household income level $\frac{R}{w} = 0.22$. The rent to household income data, the maintenance costs are taken for the median household in the 2010-2017 American Community Survey.

Comparative statics

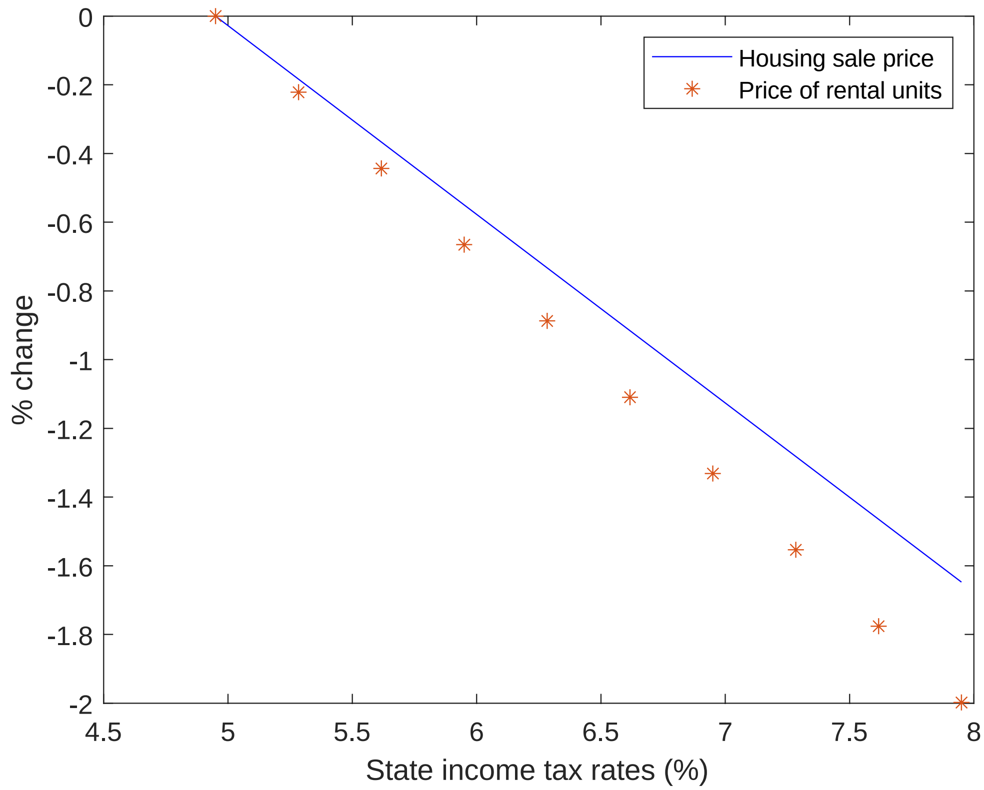
In this section, we highlight the steady-state adjustment to tax rates.

What are the effects of taxes on prices?

Income taxes lower the surplus. This causes demand for housing services to decrease. The result is vacancies take longer to sell (time on market increases), housing prices fall and the number of vacancies increases relative to the number of potential buyers (see fig. 1)

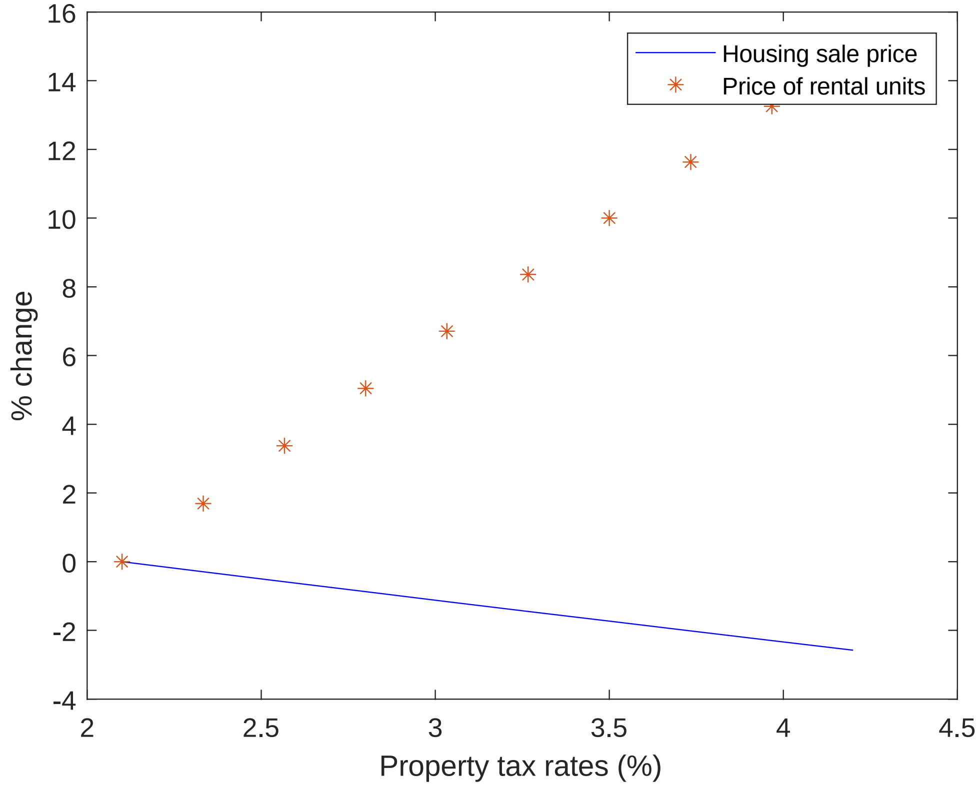
[1] Suppose that $V^b > 0$, then buyers would enter market causing p^d to decrease until $V^b = 0$.

Figure 1: the negative effect of state income taxes on housing prices



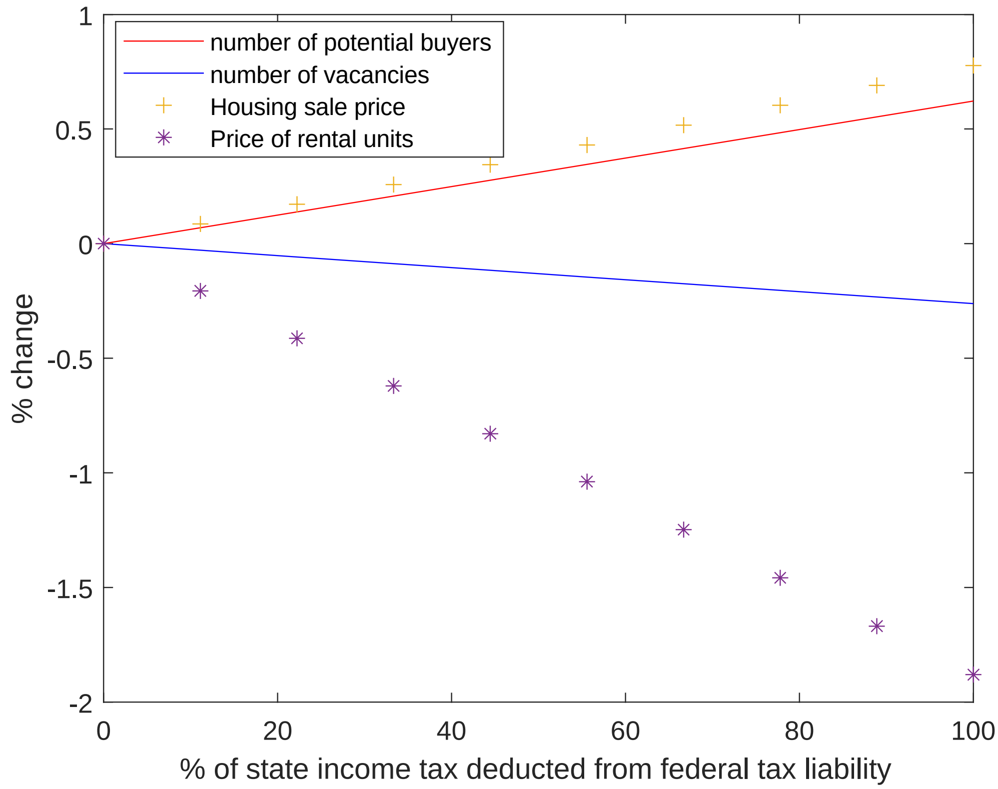
Property taxes also lower the surplus. This causes demand for housing services to decrease. The result is vacancies take longer to sell (time on market increases), housing prices fall, and the number of vacancies increases relative to the number of potential buyers (see fig. 2). The price of rental units increases since landlords are assumed to pass on the cost to renters.

Figure 2: the negative effect of property taxes on housing prices



Deductible state and local taxes raise the surplus. This causes demand for housing services to increase. The result is vacancies that take less time to sell (time on market decreases), housing prices increase and the number of vacancies decreases relative to the number of potential buyers (see fig. 3). The price of rental units falls because landlords face lower costs.

Figure 3: the effects of state and local tax deductibility


number of potential buyers
number of vacancies
Housing sale price
Price of rental units
% change
% of state income tax deducted from federal tax liability
1
0.5
0
-0.5
-1
-1.5
-2
0
20
40
60
80
100